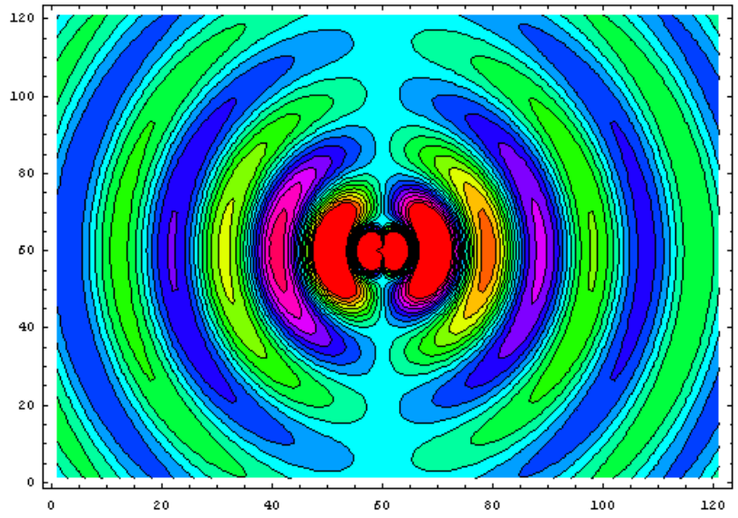
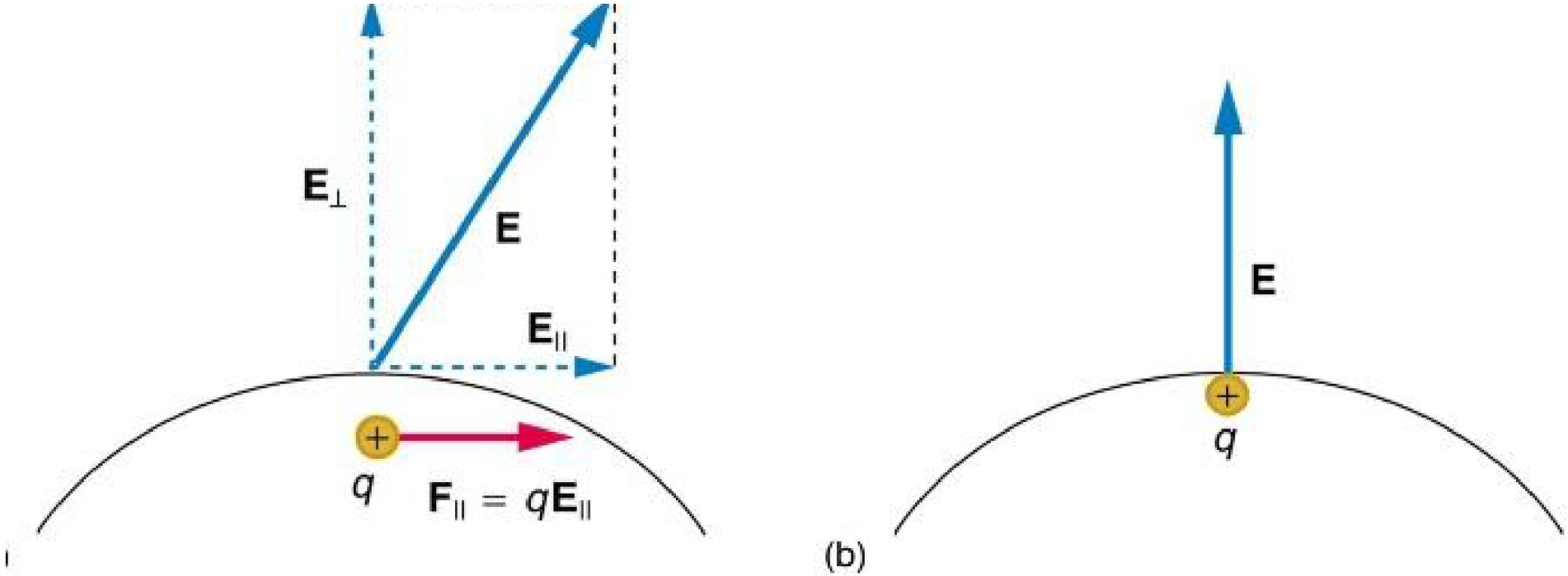
OET 1

Predavanja 3 i 4





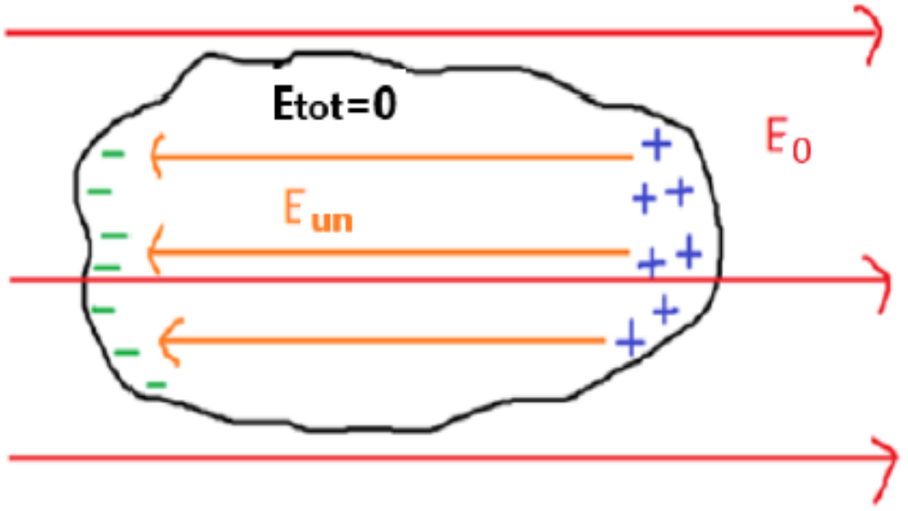
Zamislimo da na površini provodnika postoji električno polje koje ima tangencijalnu komponentu Et.

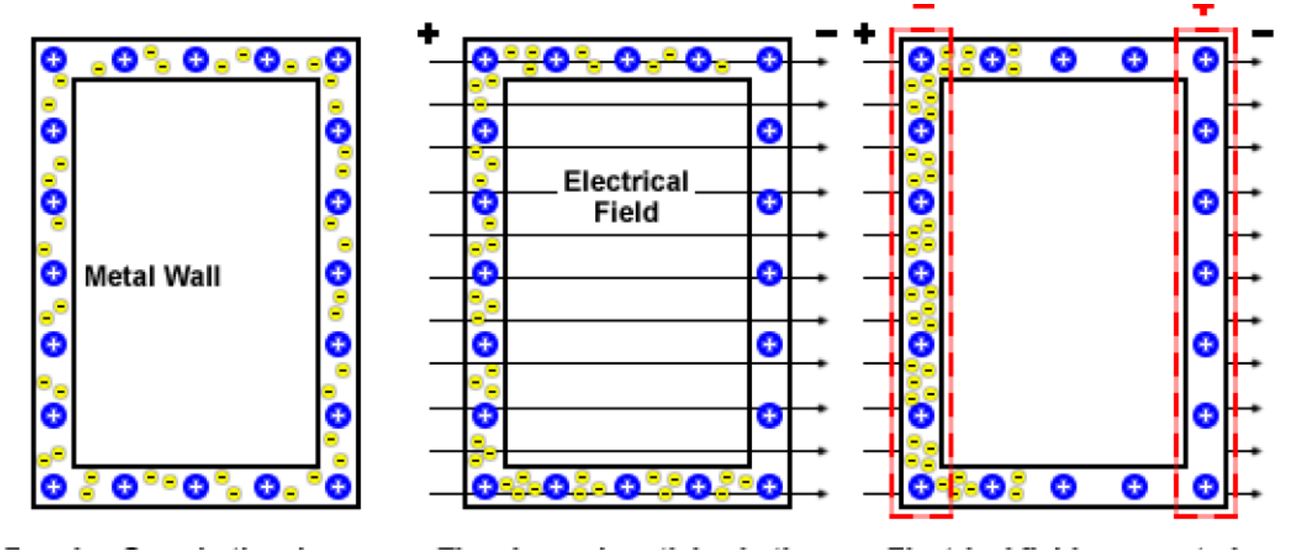
1.Na slobodne elektrone deluje tangencijalna sila F=qE

2.Elektroni počinju da se kreću duž površine i preraspodeljuju se.

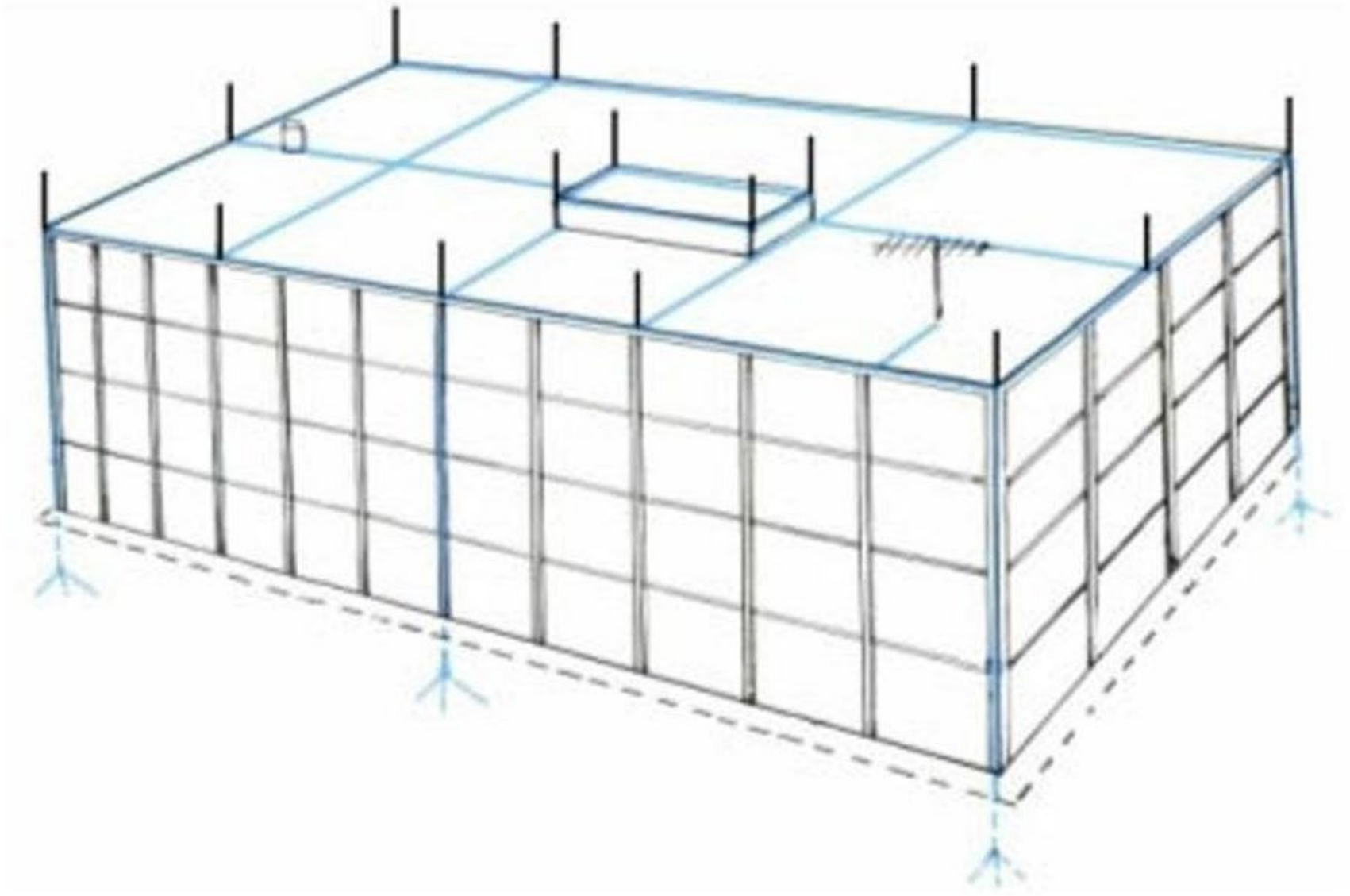
3.Njihovo novo raspoređivanje menja električno polje.

4.Taj proces traje dok se tangencijalna komponenta ukupnog električnog polja ne poništi.


Etot=0
E0
Eun

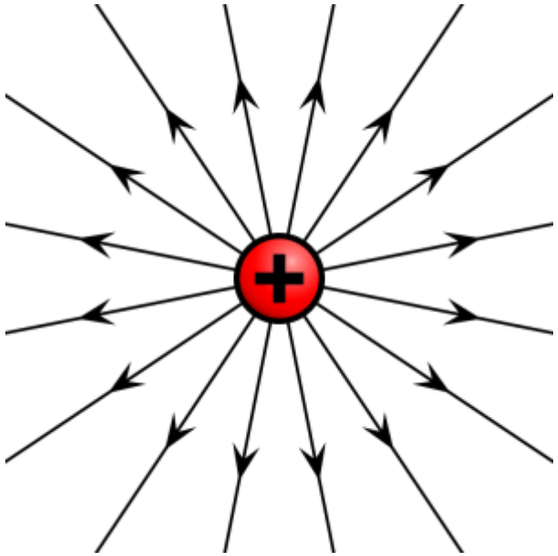

Metal Wall
Electrical Field



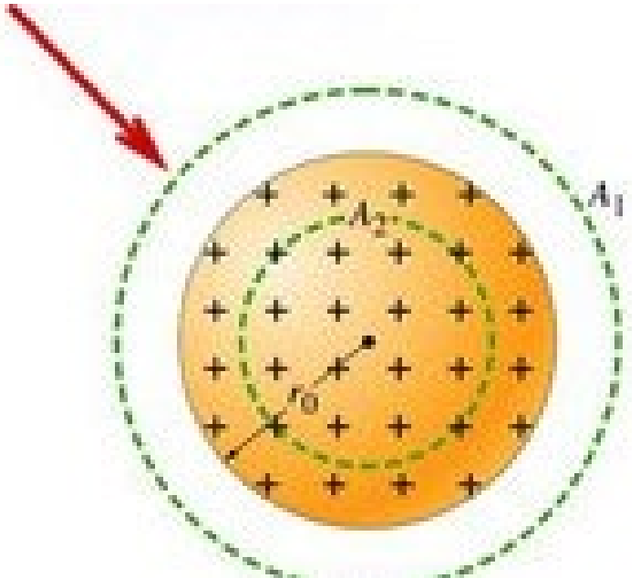



EXIT

- TELO KONAČNIH DIMENZIJA v.s Tačkasto naelektrisanje



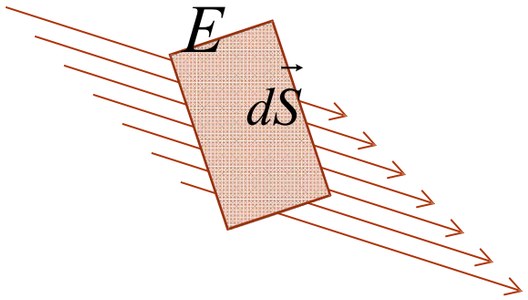




Fluks vektora jačine električnog polja

Naelektrisanje nije ravnomerno raspoređeno ako površina nije pravilnog oblika.

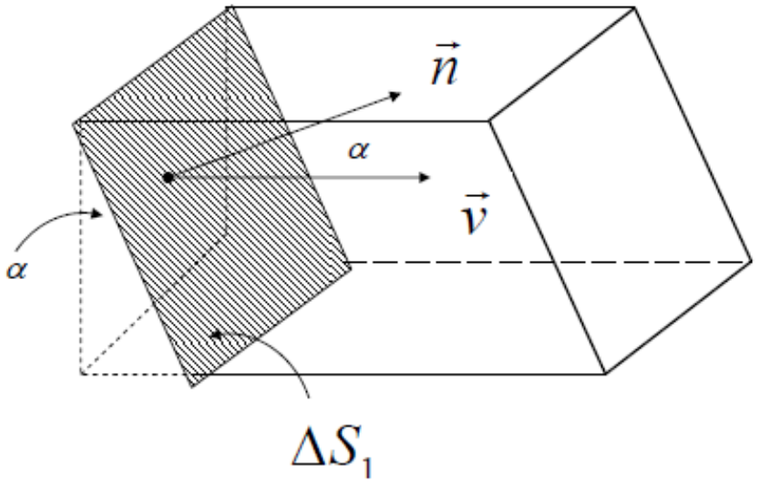
- Fluks!!!
- Posledica raspodela naelektirsanja na realnim povrsima
- Linije električnog polja i njihova gustina!!!!

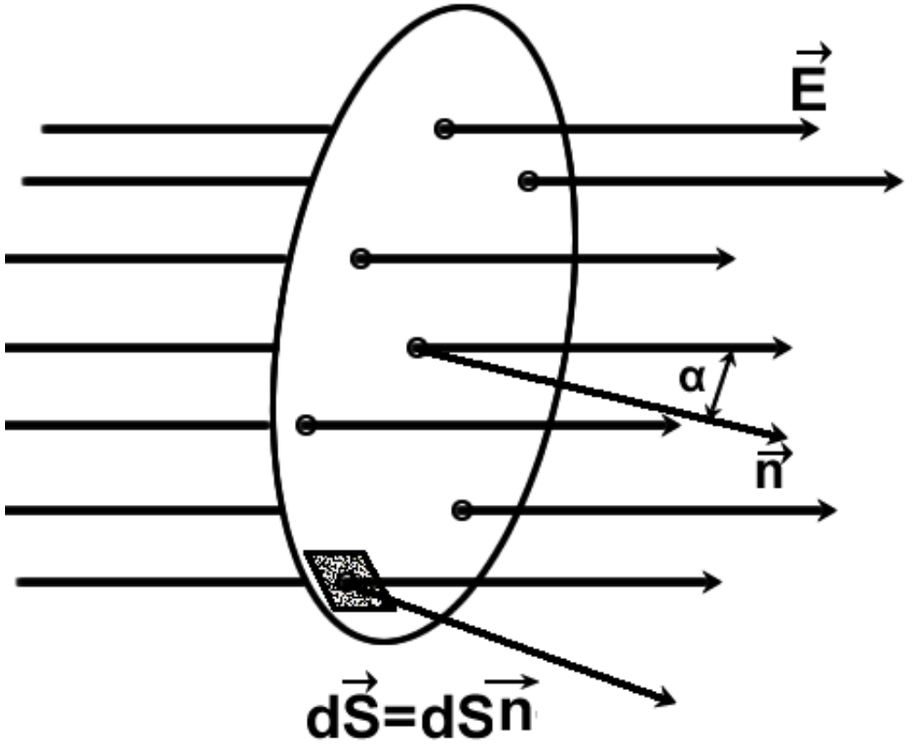


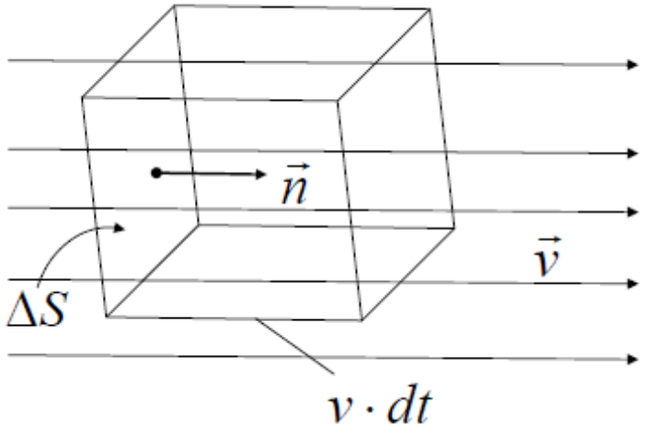
Veber -Wb

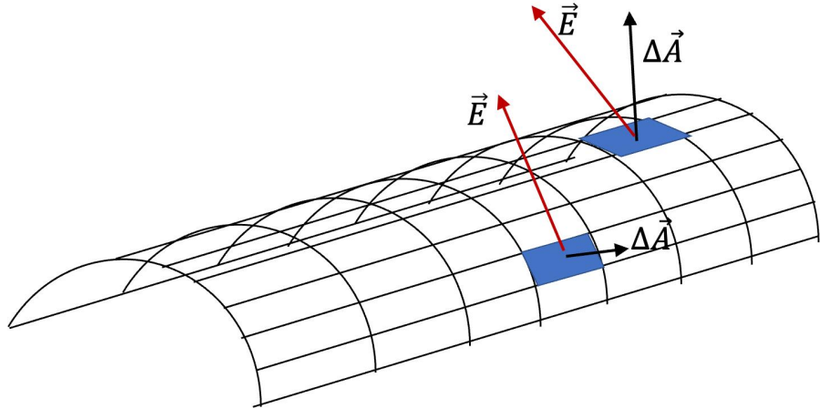
$$\Phi = \vec{E}\vec{S} = ES\cos\angle(\vec{E},\vec{S})$$

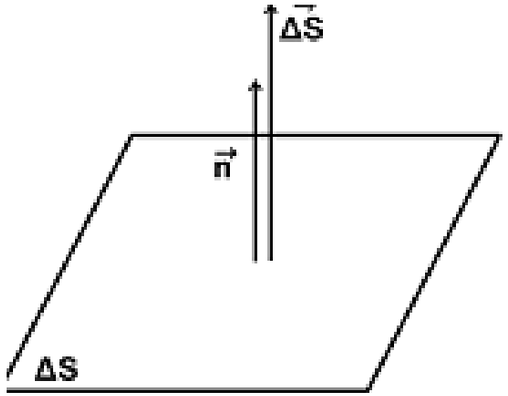
$$\Phi = \vec{E}\vec{S}$$


$\vec{n}$
α
$\vec{v}$
α
ΔS_1

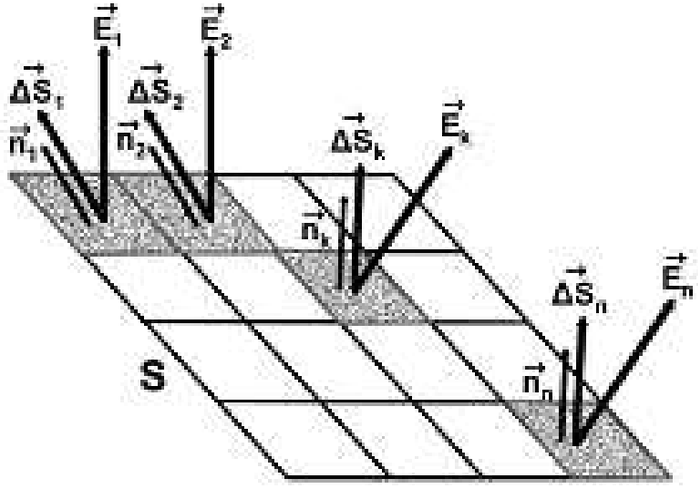

$\vec{E}$
α
$\vec{n}$
$d\vec{S}=dS\vec{n}$


$\vec{n}$
$\vec{v}$
ΔS
$v \cdot dt$


$\vec{E}$
$\Delta\vec{A}$
$\vec{E}$
$\Delta\vec{A}$



$$\Phi = \vec{E}_1 d\vec{S}_1 + \vec{E}_2 d\vec{S}_2 + \vec{E}_3 d\vec{S}_3 + \ldots..$$



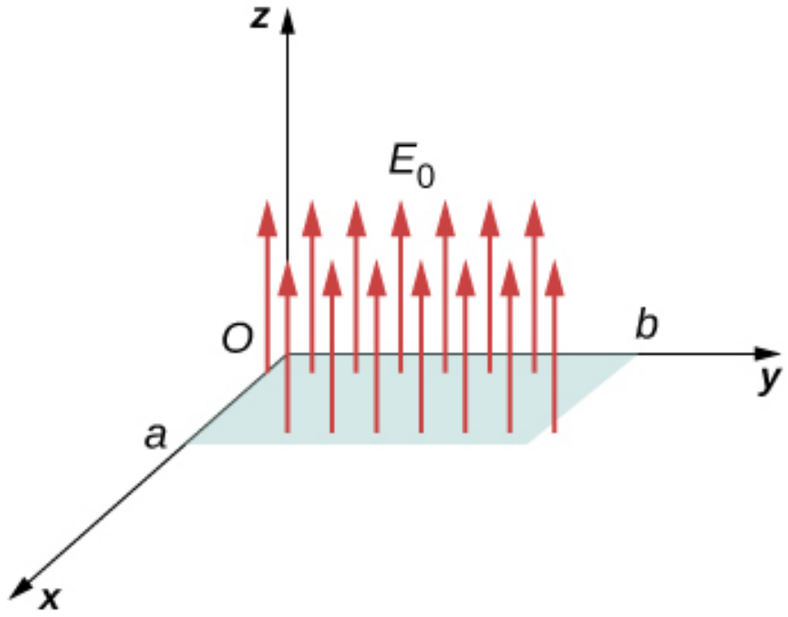
Fluks je algebarska veličina, tj. vrednost fluksa može biti pozitivna ili negativna brojka. To zavisi od toga kako je usmerena normala na površ, kroz koju se fluks izračunava.

Kada je površina zatvorena, fluks se računa u odnosu na normalu usmerenu iz površi u spoljašnjost (“spoljašnja normala”, “izlazni fluks”).

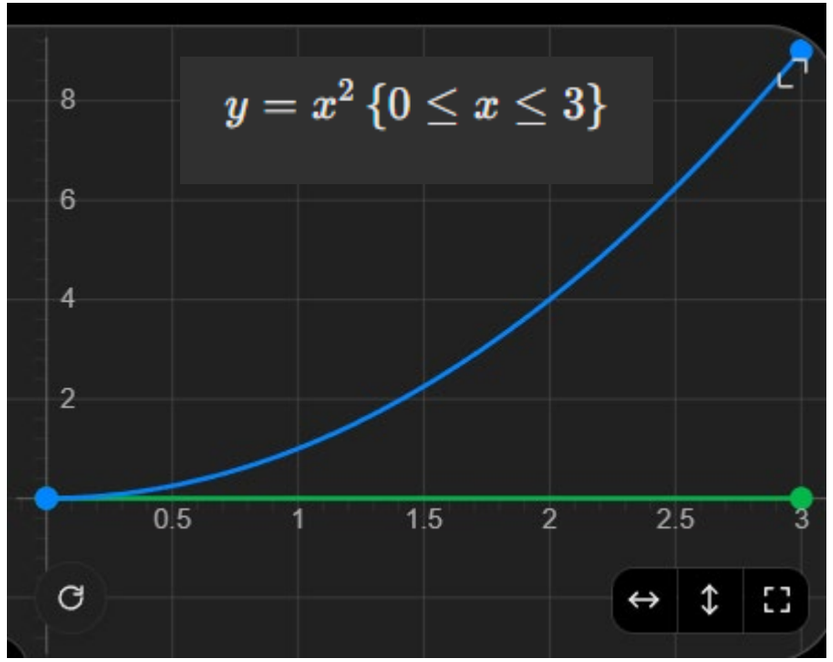
Izračunavanje fluksa kroz površi proizvoljnog oblika je veoma složen zadatak, čak i kad je polje vektora jednostavno – na primer homogeno.

This Photo by Unknown Author is licensed under CC BY

Najjednostavnije je izračunati fluks kroz površ koju linije polja samo tangiraju. Kako je tada ugao između vektora polja i normale na površ jednak 90^0, fluks kroz površ će biti jednak nuli.

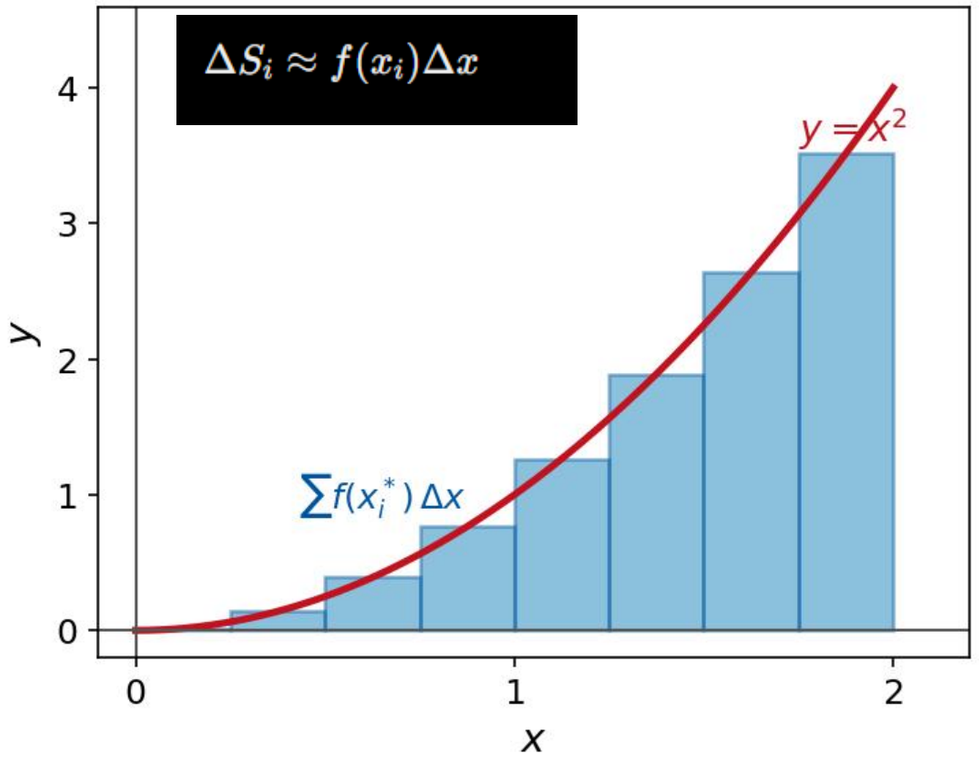


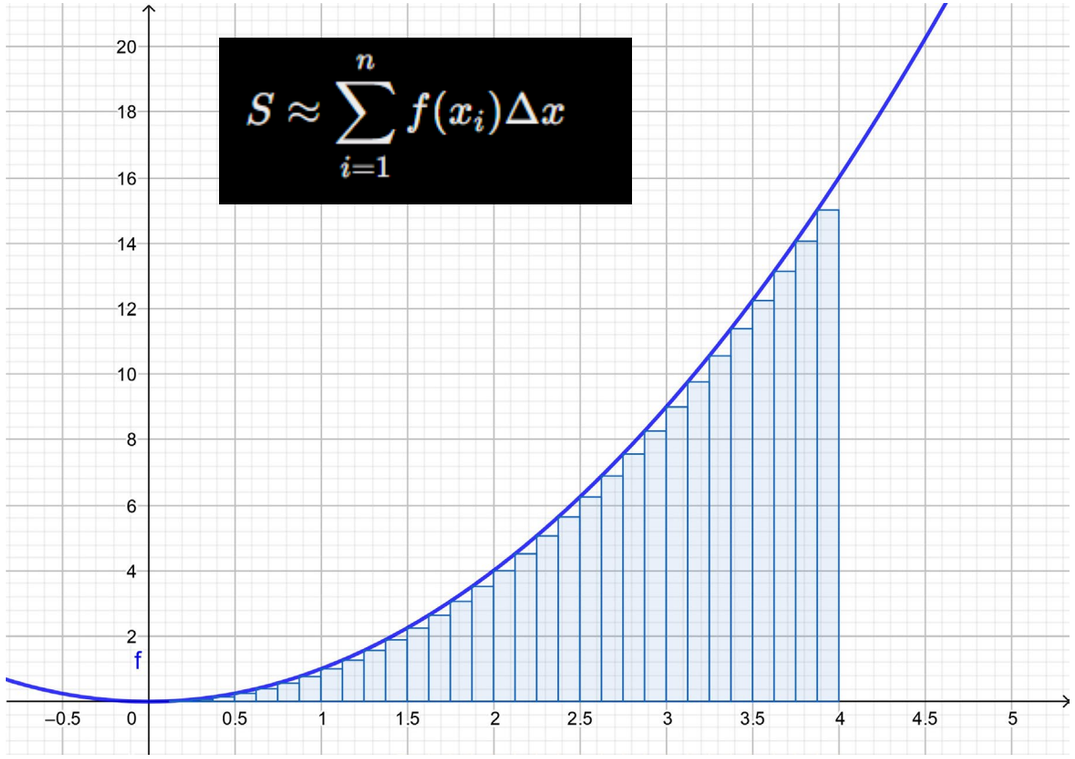
Kratak uvod u integrale

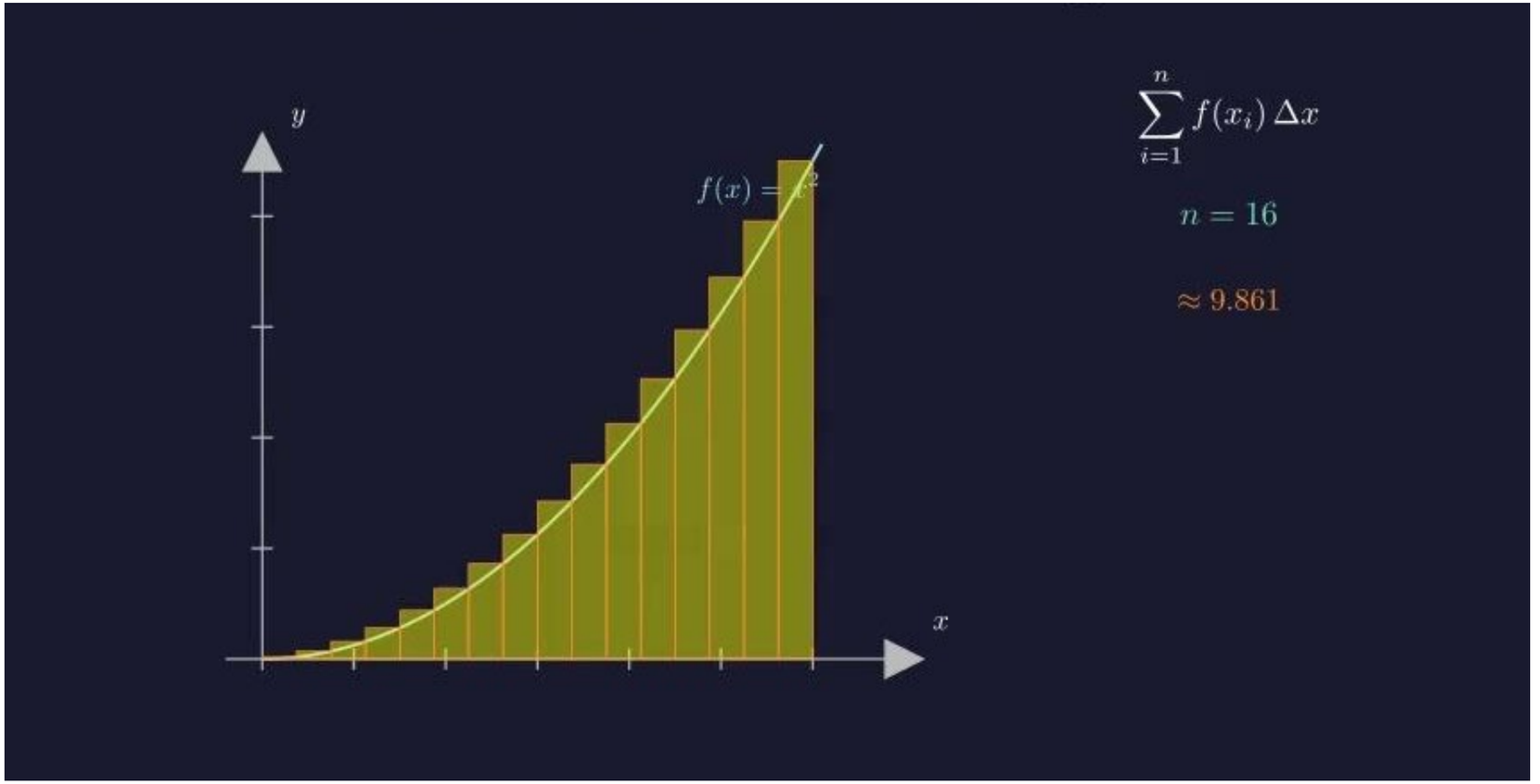


Izračunati površinu između krive, x-ose i granica x=0 i x=3.

Površinu delimo na uske pravougaonike.
Širina svakog pravougaonika je Δx,
Visina približno odgovara vrednosti funkcije.


$\Delta S_i \approx f(x_i)\Delta x$
$y = x^2$
$\sum f(x_i^*)\,\Delta x$
y
x
0
1
2
3
4


$S \approx \sum_{i=1}^{n} f(x_i)\Delta x$
f
20
18
16
14
12
10
8
6
4
2
−0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



Kada širina pravougaonika teži nuli, dobijamo tačnu površinu:

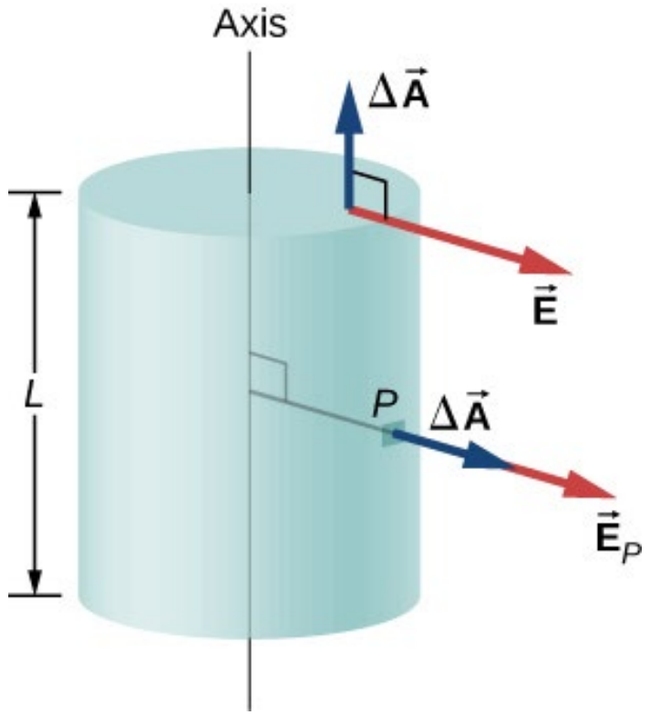
$$S = \int_0^3 x^2\,dx$$

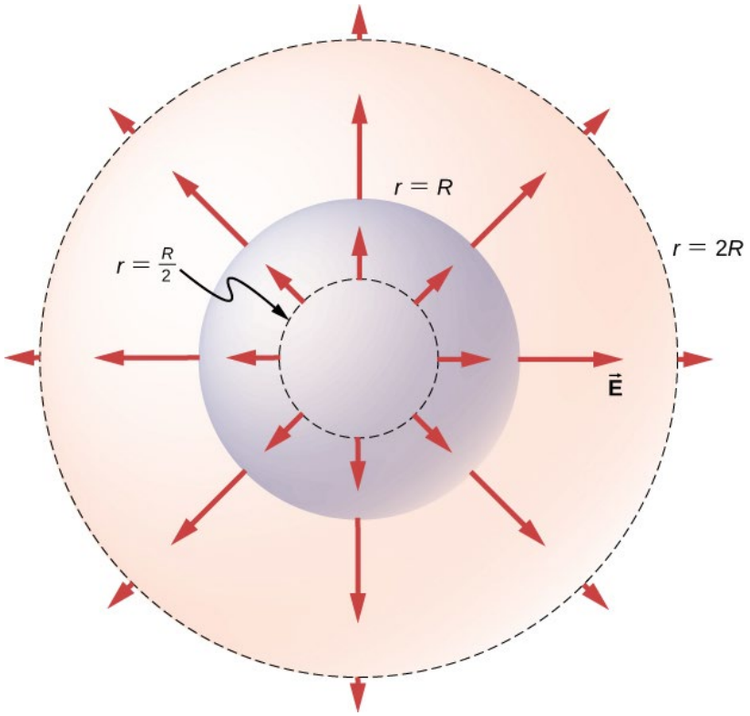
$$S = \frac{x^3}{3}\Big|_0^3 = \frac{27}{3} - 0 = \boxed{9}$$

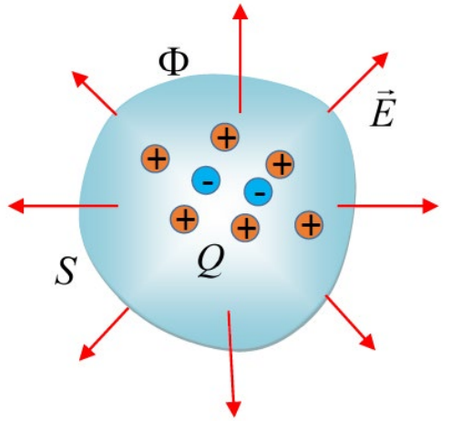
Pravilo	Formula
Množenje konstantom	$\int kf(x)\,dx = k\int f(x)\,dx$
Zbir funkcija	$\int (f+g)\,dx = \int f\,dx + \int g\,dx$
Razlika funkcija	$\int (f-g)\,dx = \int f\,dx - \int g\,dx$
Određeni integral	$\int_a^b f(x)\,dx = F(b) - F(a)$

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
2. $\int \frac{dx}{x} = \ln|x| + C$
3. $\int \sin x\,dx = -\cos x + C$
4. $\int \cos x\,dx = \sin x + C$

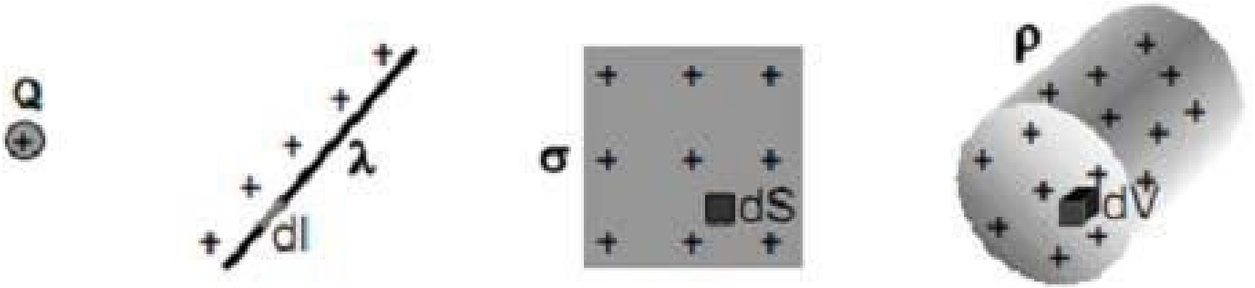
Gausov zakon

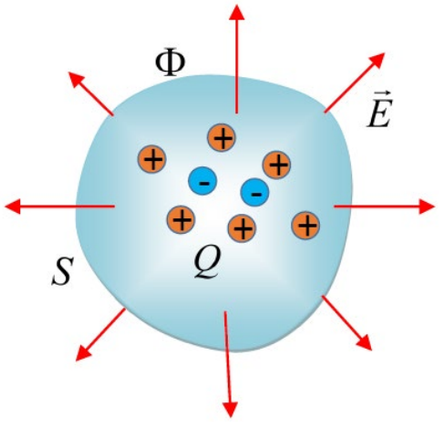


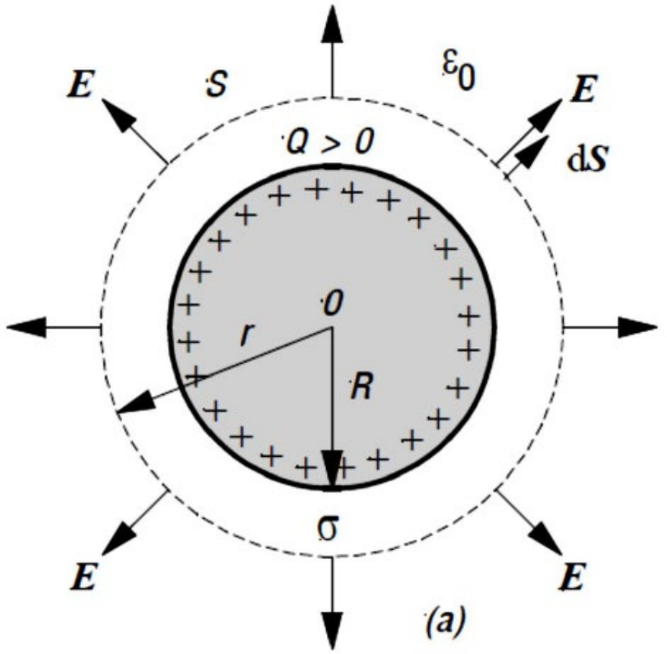




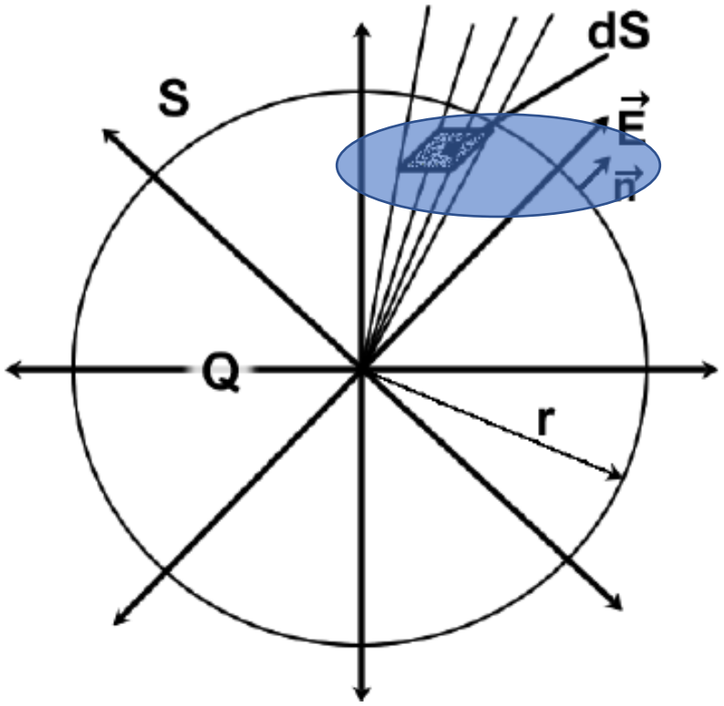
- Simetrija
- Ravnomerna raspodela nalektrisanja
- sredina







$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\vec{r}_o$$



$$\vec{E}\cdot\vec{ds} = \vec{E}\cdot ds\vec{n} = Eds\cos\left(\vec{E},\vec{n}\right) = Eds$$

- Polje ravnomenro naelektrisane sfere

$$\oint \vec{E}\, d\vec{S} = \frac{\sum Q_i}{\varepsilon_0}$$

$$\vec{E}\oint_s dS = \frac{\sum_{i\,i} Q}{\varepsilon_0}$$

$$\oint_s dS = S$$

$$\sum_i Q_i = Q$$

$$\Phi = ES = ES\cos\angle(E,S)$$

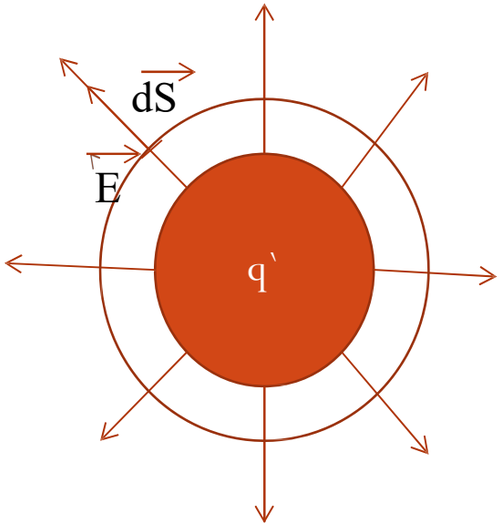
$$d\Phi = \vec{\mathrm{E}}d\vec{S}$$

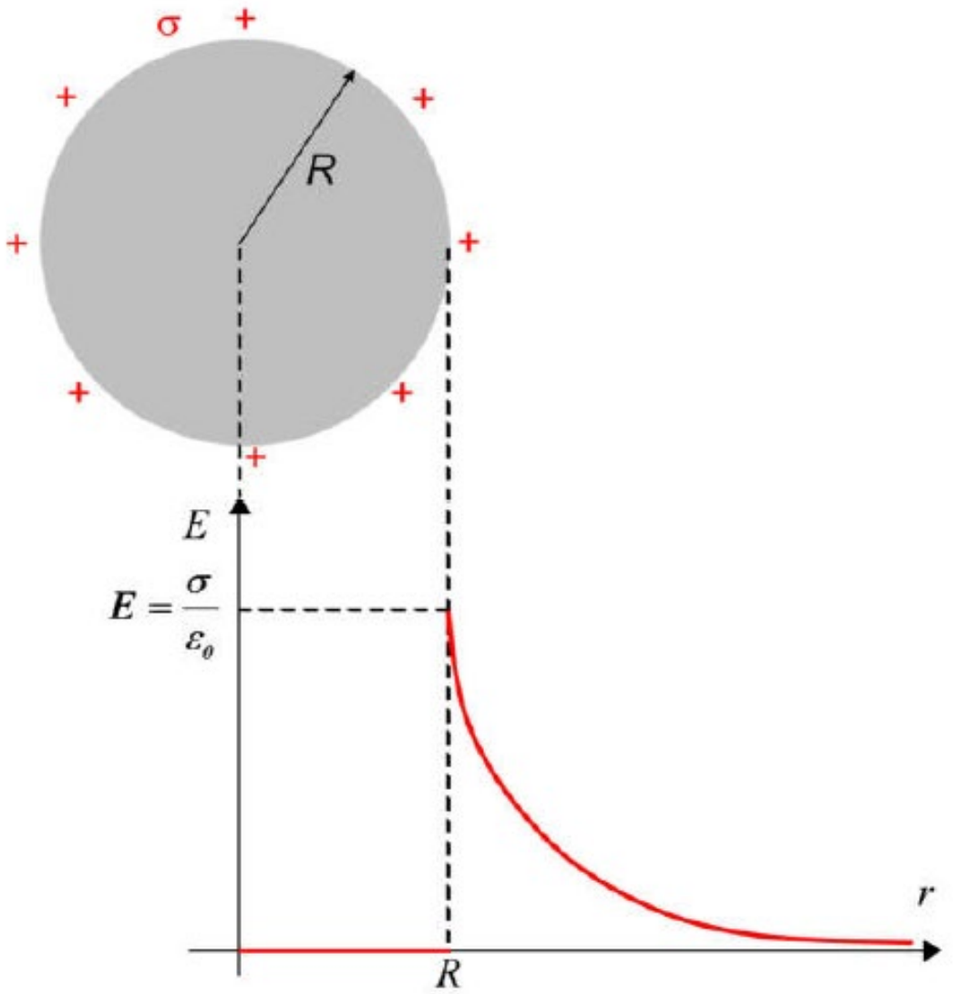
$$E\oint_s ds = \frac{\sum_i Q_i}{\varepsilon_0}$$

$$\oint_s dS = 4r^2\pi$$

$$E\,4r^2\pi\vec{r} = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{r}$$




σ
R
E
$E = \frac{\sigma}{\varepsilon_0}$
r
R

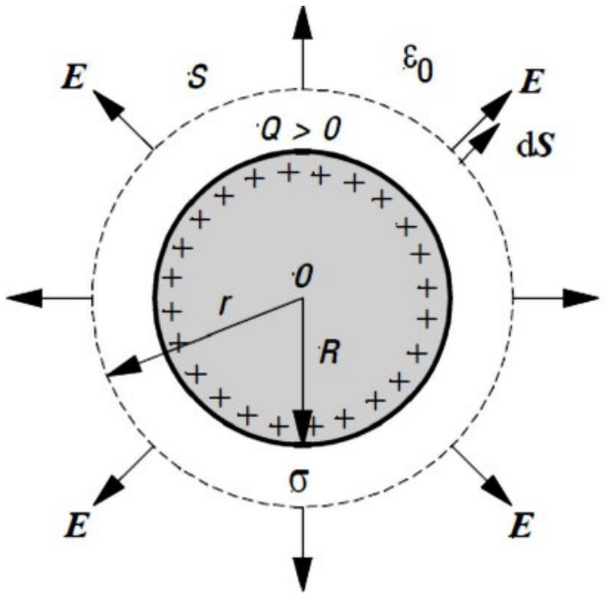
Potencijal površinski naelektrisane metalne sfere $r = \infty$

r< R $\qquad E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $\qquad \sigma = \frac{Q}{4\pi R^2}$

$$\varphi = \int_A^R \vec{E}d\vec{r} = \frac{Q}{4\pi\varepsilon_0}\int_{r_A}^{r_R}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Bigg|_{r_A}^{r_R=\infty} = \frac{Q}{4\pi\varepsilon_0 r_A} = \frac{\sigma 4\pi R^2}{4\pi\varepsilon_0 r_A} = \frac{\sigma R^2}{\varepsilon_0 r_A}$$

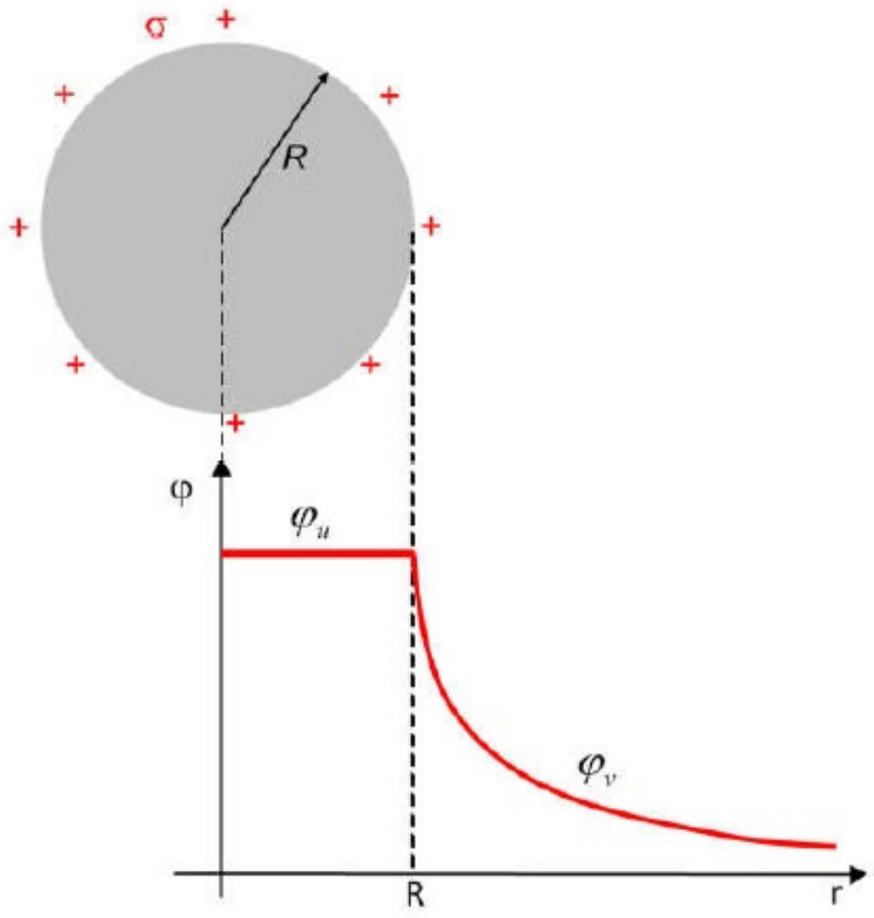
r= R

$$\varphi = \frac{\sigma R^2}{\varepsilon_0 R} = \frac{\sigma R}{\varepsilon_0}$$



$r < R$

$$\varphi = \int_{r_A}^{R} \vec{E}\,d\vec{r} + \int_R^{\infty} \vec{E}d\vec{r} = 0 + \frac{Q}{4\pi\varepsilon_0}\int_R^{\infty}\frac{dr}{r^2} = V(R)$$


σ
R
φ
φ_u
φ_v
R
r

- Polje zapreminski ravnomerno naelektrisane sfere

$$\oint \vec{E}\, d\vec{S} = \frac{\sum Q_i}{\varepsilon_0} \qquad \rho = \frac{Q}{V}$$

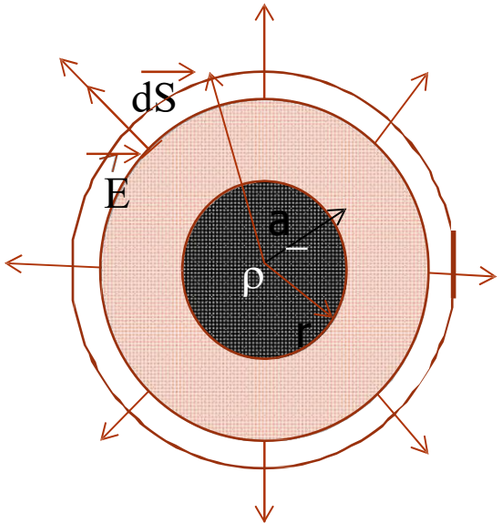
$$\vec{E} \oint_s dS = \frac{\sum_i Q_i}{\varepsilon_0} \qquad d\rho = \frac{dQ}{dV}$$

r≤a

$$\vec{E} \oint dS = \frac{\sum_i \rho dV}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0}$$

$$\vec{E}\, 4\,\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0} \mathrm{r}$$



$$\oint \vec{E}\, d\vec{S} = \frac{\sum Q_i}{\varepsilon_0}$$

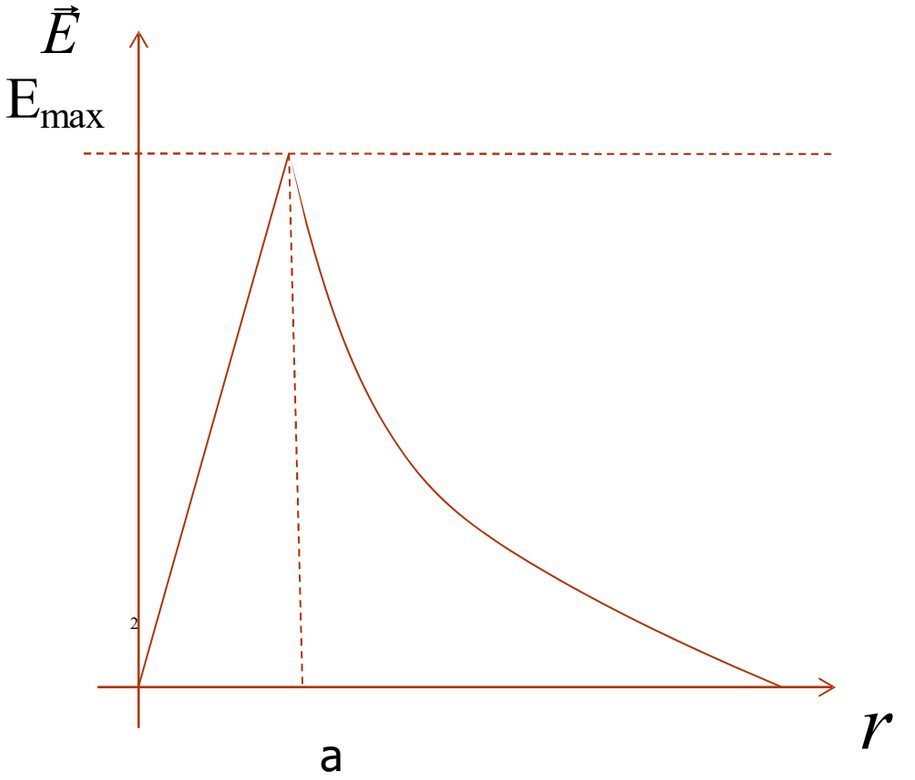
$$\vec{E} \oint_S dS = \frac{\sum_i Q_i}{\varepsilon_0}$$

r≥a

$$\vec{E}\, 4\pi r^2 = \frac{\rho \frac{4}{3}\pi a^3}{\varepsilon_0}\vec{r}$$

$$\vec{E} = \frac{\rho a^3}{3\pi r^2 \varepsilon_0}\vec{r}$$

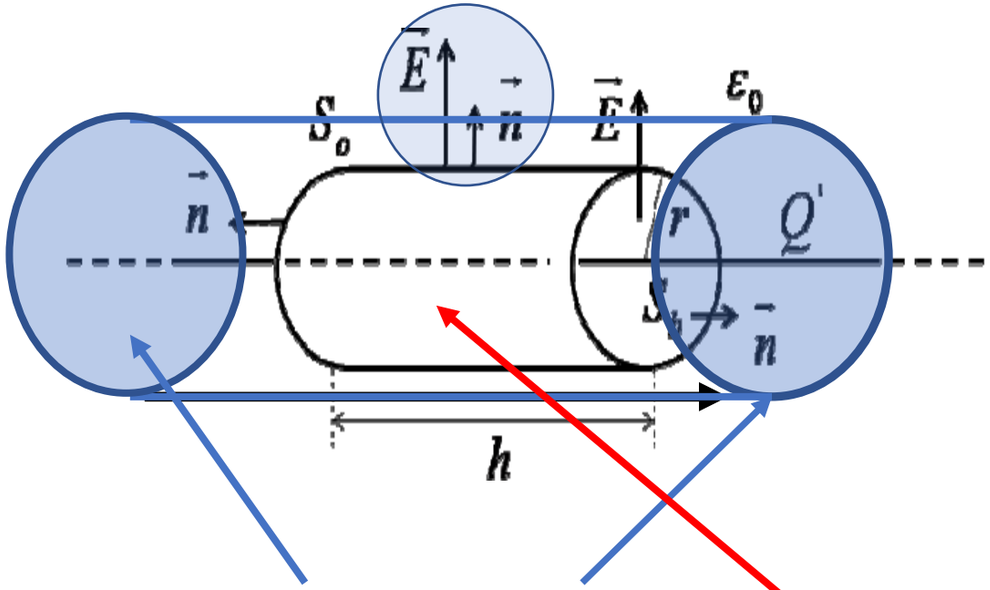
$$\vec{E} = \frac{Q}{4\pi\varepsilon_o r^2}\vec{r}$$



$$\vec{E} = \begin{cases} r \le a \\ \\ r \ge a \end{cases}$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\mathrm{r}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_o r}$$



- ✓ Linije električnog polja
- ✓ Zamišljene površine
- ✓ Fluks vektora jačine polja

$$\oint_S \vec{E}d\vec{s} = \int_{S_b} \vec{E}d\vec{s} + \int_{S_b} \vec{E}d\vec{s} + \int_{S_o} \vec{E}d\vec{s} = 0 + 0 + \int_{S_o} Eds = E\int_{S_o} ds = E2\pi rh$$

Φ_b Φ_b Φ_0

$$\Phi = \vec{E}\vec{S}cos \measuredangle (\vec{E}\,\vec{S})$$

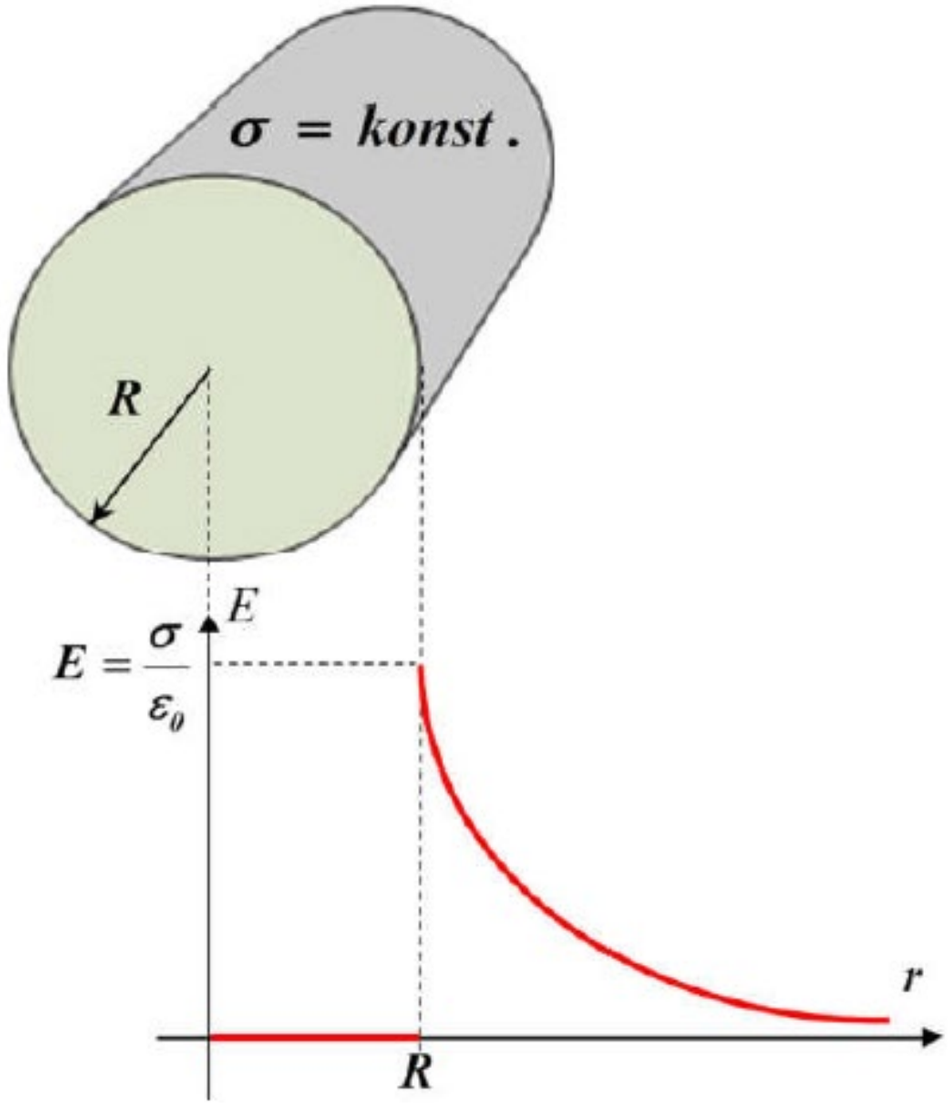
$$\Phi_{b1} = \Phi_{b1} = ES_b = 0$$

?

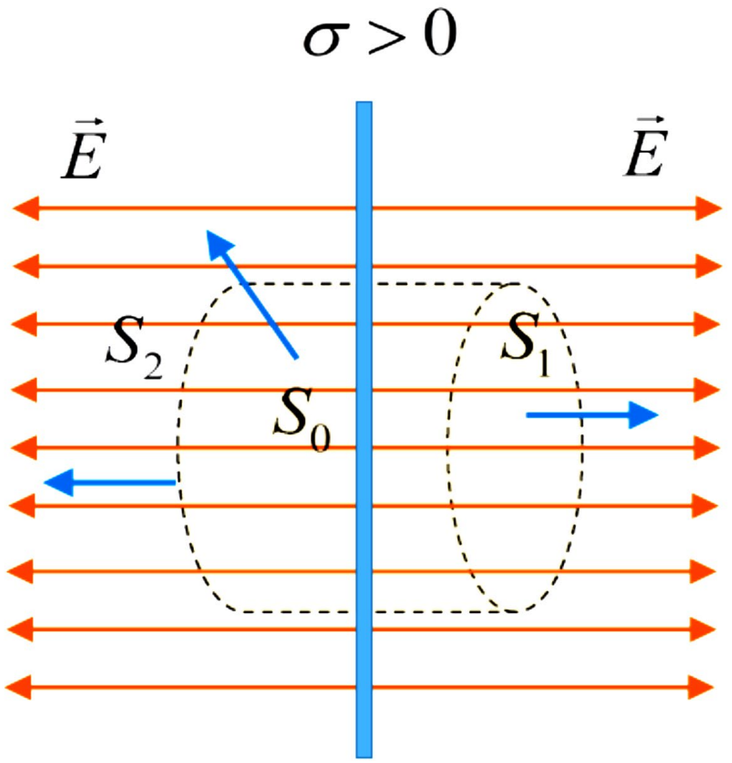
$$\Phi_0 = \vec{E}\vec{S}cos \measuredangle (90^0)$$

$$E = \frac{Q'}{2\pi\varepsilon_0 r}$$

Polje provodnog cilindra


σ = konst .
R
E
E = σ / ε0
r
R

POVRŠINSKI NAELEKTRISANA BESKONAČNA RAVAN



$$Q = \sigma S\,, \quad S_1 = S_2 = S$$

$$E_1 = E_2 = E$$

Za zatvorenu površ S usvajamo valjak simetrično postavljen u odnosu na naelektrisanu ravan.

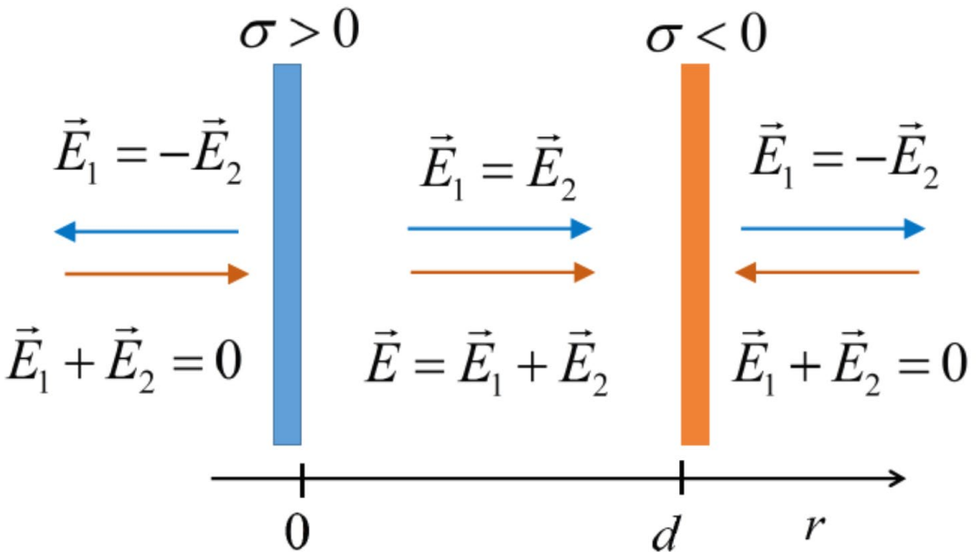
$$\oint_S \vec{E}d\vec{S} = \int_{S_0} \vec{E}d\vec{S}_0 + \int_{S_1} \vec{E}d\vec{S}_1 + \int_{S_2} \vec{E}d\vec{S}_2$$

$$= \int_{S_1} EdS_1 + \int_{S_2} EdS_2$$

$$= 0 + ES_1 + ES_2 = 2ES$$

$$2ES = \frac{Q}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0}$$

Dve površinski naelektrisane beskonačne ravani sa suprotnim naelektrisanjima



$$E = E_1 + E_2 = 2E_1 = 2\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

Potencijal polja dve naelektrisane ravani

$$E = \frac{\sigma}{\varepsilon_0},$$

$$\varphi = \int_A^R \vec{E} d\vec{r} = \int_A^R E dr = \int_{r_A}^{r_R} \frac{\sigma}{\varepsilon_0} dr = \frac{\sigma}{\varepsilon_0} \int_{r_A}^{r_R} dr = \frac{\sigma}{\varepsilon_0} \left(\overset{?}{r_R} - r_A \right)$$

